

Model-Independent Predictions for Low Energy Isoscalar Heavy Baryon Observables in the Combined Heavy Quark and Large N_c Expansion.

Z. Aziza Baccouche, Chi-Keung Chow, Thomas D. Cohen and Boris A. Gelman
Department of Physics, University of Maryland, College Park, MD 20742-4111.

Model-independent predictions for excitation energies, semileptonic form factors and electromagnetic decay rates of isoscalar heavy baryons and their low energy excited states are discussed in terms of the combined heavy quark and large N_c expansion. At leading order, the observables are completely determined in terms of the known excitation energy of the first excited state of Λ_c . At next-to-leading order in the combined expansion all heavy baryon observables can be expressed in a model-independent way in terms of two experimentally measurable quantities. We list predictions at leading and next-to-leading order.

A number of nonperturbative expansions have been used to study low-energy hadronic observables. The heavy quark expansion has been very successful in describing hadrons containing one heavy quark. In the heavy quark limit the heavy quark spin-flavor decouples from the dynamics and hadronic spectrum exhibits an approximate heavy quark spin-flavor symmetry. Heavy quark effective theory (HQET) is a consistent framework to study the $1/m_Q$ corrections to the heavy quark limit [1–6]. Another useful limit is the large N_c limit where the number of colors N_c of the QCD gauge group is taken to be large [7,8]. In the large N_c limit the baryon sector of QCD exhibits an approximate light quark spin-flavor symmetry; *eg.* as N_c goes to infinity the nucleon and Δ become degenerate [9–13]. For heavy baryons, *i.e.* baryons containing one (c or b) heavy quark it is useful to consider a double limit and a combined heavy quark and large N_c expansion [14]. In Ref. [15–17] isoscalar heavy baryons have been considered near the combined limit.

The combined heavy quark and large N_c expansion is formulated in terms of the counting parameter $\lambda \sim \Lambda_{had}/m_Q$, $1/N_c$ with $\Lambda_{had} N_c/m_Q$ arbitrary (here Λ_{had} is a typical hadronic scale). As λ approaches zero, the heavy baryon spectrum exhibits an approximate dynamical symmetry—a contracted $O(8)$ symmetry—which connects orbitally excited states of the heavy baryons to the ground state [14,15]. An effective theory describing low-energy excitations of heavy baryons can be developed using the counting rules consistent with a contracted $O(8)$ symmetry [16]. The purpose of the present paper is to use the formalism of Ref. [17] to derive model-independent relations among observables characterizing isoscalar heavy baryons and their decays. At present, experimental data is available only for the excitation energies of the doublet of the first excited state of Λ_c . At leading order in the combined expansion this energy completely determines the excitation energy of the low-energy orbitally excited state of Λ_b as well as the dominant semileptonic form factors and electromagnetic decay rates. Such predictions are necessarily rather crude as our expansion is in powers of $\lambda^{1/2}$. At next-to-leading order an additional observable is required to make further model independent predictions. As data for more heavy baryon observables (such as the excitation energy of Λ_b baryon, semileptonic and radiative decay rates of Λ_c and Λ_b baryons and their excited states) become available, the next-to-leading order predictions can be tested against experimental results. In this paper we discuss which observables are most sensitive to the NLO corrections, and, thus, can be used to extract two model-independent constants which arise at leading and next-to-leading order in the effective expansion. Using the formalism of Ref. [17] we derive model independent relations among a number of observables characterizing isoscalar heavy baryons and their decays.

The effective degrees of freedom near the combined limit are the collective motion of the brown muck—the light quarks and gluons—relative to the heavy quark. As shown in Ref. [16], the energy of the collective excitations is of order $\lambda^{1/2}$ while the energy of the intrinsic excitations of the brown muck itself are of order λ^0 . The effective operators that excite the collective degrees of freedom can be defined from the corresponding QCD operators in the combined limit. The construction of these operators from QCD in a model-independent way was discussed in detail in Ref. [15,16]. They are the total momentum \vec{P} (a constant of the motion) and its conjugate position operator \vec{X} of the entire system. These operators commute up to corrections of order λ with another pair of the collective conjugate operators \vec{p} and \vec{x} which are related to the momentum and position of the brown muck relative to heavy quark.

We have so far focused on the isoscalar heavy baryons, Λ_c and Λ_b , and their excited states. The effective Hamiltonian for these states can be written solely in terms of the collective operators discussed above. As shown in Ref. [15], the contracted $O(8)$ symmetry naturally arises in the combined limit if the counting rules of the collective operators are given by,

$$(x, X, X_Q, X_\ell) \sim \lambda^{1/4},$$

$$(p, P, P_Q, P_\ell) \sim \lambda^{-1/4}, \quad (1)$$

in the sense that the typical matrix elements of these operators between the low-lying heavy baryon states scale as $\lambda^{1/4}$ and $\lambda^{-1/4}$. The effective Hamiltonian based on these counting rules up to terms of order λ was derived in Ref. [16]:

$$H_{\text{eff}} = \underbrace{(m_H + m_N)}_{\mathcal{H}_{\lambda^{-1}}} + \underbrace{c_0}_{\mathcal{H}_{\lambda^0}} + \underbrace{\left(\frac{P^2}{2(m_N + m_H)} + \frac{p^2}{2\mu_Q} + \frac{1}{2}\kappa x^2\right)}_{\mathcal{H}_{\lambda^{1/2}}} + \underbrace{\frac{1}{4!}\alpha x^4}_{\mathcal{H}_{\lambda^1}} + \mathcal{O}(\lambda^{3/2}), \quad (2)$$

where m_N is the nucleon mass; m_H is the mass of the heavy meson containing the same heavy quark as the heavy baryon. These masses are measurable quantities while the corresponding brown muck and the heavy quark masses are not. In the combined limit, the difference between the heavy quark and the corresponding heavy meson mass is of order λ^0 ; the difference between the brown muck mass and the nucleon mass is of order λ^0 as well. The heavy meson and the nucleon masses scale as λ^{-1} in the combined limit [16]. The reduced mass, μ_Q , which is of order λ^{-1} , is given by,

$$\mu_Q = \frac{m_N m_H}{m_N + m_H}. \quad (3)$$

In eq. (2), \mathcal{H}_{λ^n} refers to the piece of the Hamiltonian whose contribution is of order λ^n . The term $\mathcal{H}_{\lambda^{1/2}}$ is referred to as leading order (LO), and \mathcal{H}_λ as next-to-leading order (NLO). The combined expansion is in powers of $\lambda^{1/2}$.

The effective Hamiltonian in eq. (2) contains three phenomenological parameters— c_0 , κ and α which are of order λ^0 . At order λ^0 these constants are independent of the heavy quark flavor. However, they contain $1/m_Q \sim \lambda$ corrections. As seen from eq. (2), the $1/m_Q$ corrections to κ and α contribute only at order $\lambda^{3/2}$, *i.e.* at next-to-next-to-leading order (NNLO) and higher. Hence they can be neglected when we work only up to NLO. The $1/m_Q$ correction to c_0 contributes at order λ , *i.e.* at NLO. However, the dynamics of the collective degrees of freedom does not depend on the constant c_0 which only determines overall ground state energy. Thus, only two flavor-independent constants up to NLO determine the low-energy dynamics of the effective degrees of freedom—the collective motion of the brown muck relative to the heavy quark. They can be extracted from data in a model-independent way provided at least two relevant observables are measured.

The phenomenology of the Λ_c and Λ_b baryons and their first orbitally excited states based on the effective Hamiltonian in eq. (2) was discussed in Ref. [17]. In addition to the Hamiltonian, the effective operators determining semileptonic form factors and electromagnetic decay rates have been derived in a model-independent way in the combined expansion [16]. Remarkably up to NLO, these effective operators do not contain additional phenomenological parameters. In what follows, we present the model-independent predictions of the effective theory at LO and NLO. At LO, these predictions are the same as obtained from the bound state picture of the heavy baryon in which the heavy baryon is thought of as the bound state of the heavy meson and an ordinary baryon (*eg.* the nucleon) [18–27]. However, the formalism of Ref. [17] puts these results in a model-independent framework.

A useful eigenbasis for the low-lying heavy baryons is spanned by the eigenstates of the leading order terms of the effective Hamiltonian, *i.e.* $|\Lambda_Q; N, l, m; \sigma\rangle$, where N, l, m are the quantum numbers of the three-dimensional harmonic oscillator and σ is the third component of the heavy quark spin ($\sigma = \pm 1/2$). The heavy quark spin decouples from the dynamics up to NNLO. As a result, the total angular momentum of the heavy baryon is $J = l + s_Q = |l \pm 1/2|$. The heavy baryons are the eigenstates of the total angular momentum, *i.e.* they are appropriate linear superpositions of the states $|\Lambda_Q; N, l, m; \sigma\rangle$. We denote these states by $|\Lambda_Q; N, J, J_z\rangle$. The ground states in both the charm and bottom sectors, Λ_c and Λ_b , have been observed. In addition, the doublet of the first orbitally excited state—(Λ_{c1} , Λ_{c1}^*)—has been observed [28–30]. For further convenience we use the following notation for these states:

$$\begin{aligned} |\Lambda_Q\rangle &\equiv |\Lambda_Q; 0, \frac{1}{2}, J_z\rangle \sim \Lambda_c, \Lambda_b, \\ |\Lambda_{Q1}\rangle &\equiv |\Lambda_Q; 1, \frac{1}{2}, J_z\rangle \sim \Lambda_c(2593), \Lambda_b(?), \\ |\Lambda_{Q1}^*\rangle &\equiv |\Lambda_Q; 1, \frac{3}{2}, J_z\rangle \sim \Lambda_c(2625), \Lambda_b(?). \end{aligned} \quad (4)$$

The states of the doublet are degenerate in the combined limit up to NNLO. This leads to an order λ ambiguity in the determination of the energy of the first excited state. One way to define this energy is to take the spin-averaged mass of the doublet:

$$m_{\Lambda_Q^*} \equiv \frac{1}{3}m_{\Lambda_{Q1}} + \frac{2}{3}m_{\Lambda_{Q1}^*}. \quad (5)$$

In the charm sector the mass $m_{\bar{\Lambda}_c^*}$ is approximately 2610 MeV . Similarly, the heavy meson mass, m_H , can be taken as the spin-averaged mass of the corresponding pseudoscalar and pseudovector mesons:

$$\begin{aligned} m_{\bar{D}} &\equiv \frac{1}{4}m_D + \frac{3}{4}m_{D^*} \approx 1980 \text{ MeV}, \\ m_{\bar{B}} &\equiv \frac{1}{4}m_B + \frac{3}{4}m_{B^*} \approx 5310 \text{ MeV}. \end{aligned} \quad (6)$$

The excitation energies up to NLO are given by,

$$\Delta m_c \equiv m_{\bar{\Lambda}_c^*} - m_{\Lambda_c} = \sqrt{\frac{\kappa}{\mu_c}} + \frac{5}{4!} \frac{\alpha}{\kappa \mu_c} + \mathcal{O}(\lambda^{3/2}), \quad (7)$$

$$\Delta m_b \equiv m_{\bar{\Lambda}_b^*} - m_{\Lambda_b} = \sqrt{\frac{\kappa}{\mu_b}} + \frac{5}{4!} \frac{\alpha}{\kappa \mu_b} + \mathcal{O}(\lambda^{3/2}), \quad (8)$$

where the order λ corrections are determined by treating the $\alpha x^4/4!$ term in Hamiltonian perturbatively. As indicated above, these excitation energies are completely determined in terms of two constants, κ and α .

In addition to the spectroscopic observables, the effective theory predicts the dominant semileptonic form factors and electromagnetic decay rates of the excited heavy baryons. Up to NLO these observables are completely determined in terms of κ and α . Semileptonic form factors and electromagnetic decays have yet to be measured.

As discussed in Ref. [17], the effective theory in the combined limit can reliably predict the dominant semileptonic form factors only for the transfer velocities, $|\delta \vec{v}|$, of order $\lambda^{3/4}$. However, this is the range of the velocity transfers within which the form factors significantly change from their values at zero recoil.

In the combined limit the hadronic amplitude of the $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$ decay up to NLO is given in terms of a single independent form factor [17]:

$$\langle \Lambda_c(\vec{v}') | \bar{c} \gamma^\mu (1 - \gamma_5) b | \Lambda_b(\vec{v}) \rangle = \Theta(z) \bar{u}_c(\vec{v}') \gamma^\mu (1 - \gamma_5) u_b(\vec{v}) (1 + \mathcal{O}(\lambda)), \quad (9)$$

where Dirac spinors are normalized by $\bar{u}_Q(\vec{v}, s) u_Q(\vec{v}, s) = 1$.

The form factor $\Theta(z)$ in eq. (9) is given as a function of the kinematic variable z (instead of the more usual velocity transfer $|\delta \vec{v}|$ or momentum transfer q) defined by:

$$z \equiv \frac{m_N |\delta \vec{v}|}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{1/2}}. \quad (10)$$

As a function of the velocity transfer $|\delta \vec{v}|$ the form factor Θ has an essential singularity in the combined limit, while $\Theta(z)$ is a smooth function. The new kinematic variable z is easily related to the more familiar variable—the velocity transfer parameter $w = v \cdot v'$ where v and v' are the 4-velocities of the initial and final states:

$$z \equiv \frac{m_N \sqrt{2(w-1)}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{1/2}} \left(1 + \mathcal{O}(\lambda^{3/2}) \right). \quad (11)$$

The relation in eq. (11) is valid for velocity transfers of order $\lambda^{3/4}$.

The form factor $\Theta(z)$ was calculated up to NLO in Ref. [17]:

$$\begin{aligned} \Theta(z) &= \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \exp\left(\frac{-z^2}{2\sqrt{\kappa}}\right) \\ &\left(1 + \frac{\alpha}{4! \kappa^{5/2} (\sqrt{\mu_b} + \sqrt{\mu_c})} \left(\frac{45\kappa(\sqrt{\mu_b} - \sqrt{\mu_c})^2}{16\kappa^{3/2} \sqrt{\mu_b \mu_c}} + 5z^2 \sqrt{\kappa} - \frac{1}{4}z^4 \right) \right) (1 + \mathcal{O}(\lambda^{3/2})). \end{aligned} \quad (12)$$

Useful quantities extractable from experiment are the value of $\Theta(z)$ at zero recoil and the curvature at zero recoil (the slope of $\Theta(z)$ at $z = 0$ vanishes). Up to NLO, these observables are given by [17]:

$$\Theta_0 \equiv \Theta(z=0) = \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left(1 + \frac{\alpha}{4!} \frac{45(\sqrt{\mu_b} - \sqrt{\mu_c})^2}{16\kappa^{3/2} \sqrt{\mu_b \mu_c} (\sqrt{\mu_b} + \sqrt{\mu_c})} \right) (1 + \mathcal{O}(\lambda^{3/2})), \quad (13)$$

$$\rho \equiv \frac{\partial^2 \Theta}{\partial z^2}(z=0) = -\frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{\sqrt{\kappa} (\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left(1 + \frac{\alpha}{4!} \frac{45(\sqrt{\mu_b} - \sqrt{\mu_c})^2 - 160\sqrt{\mu_b \mu_c}}{16\kappa^{3/2} \sqrt{\mu_b \mu_c} (\sqrt{\mu_b} + \sqrt{\mu_c})} \right) (1 + \mathcal{O}(\lambda^{3/2})). \quad (14)$$

At leading order, Θ_0 is independent of κ and α ; it is an overall normalization which is approximately 0.998. This value is very close to the heavy quark effective theory (HQET) value of unity, despite the fact that the present expansion is not the pure heavy quark expansion. The curvature at zero recoil and a slope at zero recoil of the form factor Θ as a function of w are related by,

$$\frac{\partial\Theta}{\partial w}(w=1) = \frac{m_N^2}{\sqrt{\mu_b} + \sqrt{\mu_c}} \rho. \quad (15)$$

Note that ρ is of order λ^0 while $\partial\Theta/\partial w(w=1)$ is of order $\lambda^{3/2}$ in the combined limit.

There are two semileptonic decay channels of Λ_b corresponding to the decays to the doublet of the first excited state: $\Lambda_b \rightarrow \Lambda_{c1}\ell\bar{\nu}$ and $\Lambda_b \rightarrow \Lambda_{c1}^*\ell\bar{\nu}$. As shown in Ref. [17], the dominant form factor which determines the hadronic amplitudes for both of these channels is given by,

$$\begin{aligned} \langle \Lambda_{c1}(\vec{v}') | \bar{c}\gamma^\mu(1-\gamma_5)b | \Lambda_b(\vec{v}) \rangle &= \sqrt{3}\Xi(z)\bar{u}_c(\vec{v}')\gamma^\mu(1-\gamma_5)u_b(\vec{v})(1+\mathcal{O}(\lambda)), \\ \langle \Lambda_{c1}^*(\vec{v}') | \bar{c}\gamma^\mu(1-\gamma_5)b | \Lambda_b(\vec{v}) \rangle &= \Xi(z)\bar{u}_{c\nu}(\vec{v}')(\sigma^{\nu\mu}\gamma_5 - g^{\mu\nu})u_b(\vec{v})(1+\mathcal{O}(\lambda)), \end{aligned} \quad (16)$$

where the Rarita-Schwinger spinors are normalized by $\bar{u}_\nu(\vec{v},s)u^\nu(\vec{v},s) = -1$.

For the velocity transfers of order $\lambda^{3/4}$ the form factor $\Xi(z)$ is determined in Ref. [17] up to NLO:

$$\begin{aligned} \Xi(z) &= \frac{4z\mu_b^{3/8}\mu_c^{5/8}}{\kappa^{1/4}(\sqrt{\mu_b}+\sqrt{\mu_c})^2} \exp\left(-\frac{z^2}{2\sqrt{\kappa}}\right) \\ &\left(1 + \frac{\alpha}{4!\kappa^{5/2}(\sqrt{\mu_b}+\sqrt{\mu_c})} \left(\frac{(105\mu_b-230\sqrt{\mu_b\mu_c}+45\mu_c)\kappa}{16\sqrt{\mu_b\mu_c}} + \frac{13}{2}\kappa^{1/2}z^2 - \frac{1}{4}z^4\right)\right) (1+\mathcal{O}(\lambda^{3/2})). \end{aligned} \quad (17)$$

At zero recoil the form factor $\Xi(z)$ vanishes. The slope of $\Xi(z)$ at zero recoil is given by,

$$\sigma \equiv \frac{\partial\Xi}{\partial z}(z=0) = \frac{4\mu_b^{3/8}\mu_c^{5/8}}{\kappa^{1/4}(\sqrt{\mu_b}+\sqrt{\mu_c})^2} \left(1 + \frac{\alpha}{4!} \frac{105\mu_b-230\sqrt{\mu_b\mu_c}+45\mu_c}{16\kappa^{3/2}\sqrt{\mu_b\mu_c}(\sqrt{\mu_b}+\sqrt{\mu_c})}\right) (1+\mathcal{O}(\lambda^{3/2})). \quad (18)$$

Radiative decays of the excited heavy baryons might also be measured. Due to the small available phase space, radiative decays are expected to have a large branching ratio for excited Λ_c and Λ_b baryons; moreover, they may be the dominant decay channels of the excited Λ_b baryons. It was shown in a model-independent way in Ref. [17], that in the combined limit the electromagnetic decays of the doublet (Λ_{Q1} , Λ_{Q1}^*) in the charm and bottom sectors are dominated by dipole radiation. The total decay rate averaged over the initial state and summed over all final states is the same for Λ_{Q1} and Λ_{Q1}^* [17]. The total decay rates in the charm and bottom sector up to NLO are found to be:

$$\Gamma(\Lambda_{c1} \rightarrow \Lambda_c \gamma) = \frac{1}{6}e^2\kappa \left(\frac{m_{\bar{D}} - m_N}{m_{\bar{D}}m_N}\right)^2 \left(1 - \frac{\alpha}{4!} \frac{5}{\sqrt{\kappa^3\mu_c}}\right) (1+\mathcal{O}(\lambda)), \quad (19)$$

$$\Gamma(\Lambda_{b1} \rightarrow \Lambda_b \gamma) = \frac{1}{6}e^2\kappa \left(\frac{m_{\bar{B}} + m_N}{m_{\bar{B}}m_N}\right)^2 \left(1 - \frac{\alpha}{4!} \frac{5}{\sqrt{\kappa^3\mu_b}}\right) (1+\mathcal{O}(\lambda)), \quad (20)$$

where e is the electromagnetic coupling constant ($e^2 \approx 1/137$). It is interesting to note that the ratio of these decay rates at LO is $\Gamma(\Lambda_{c1} \rightarrow \Lambda_c \gamma)/\Gamma(\Lambda_{b1} \rightarrow \Lambda_b \gamma) \approx 0.2$, which significantly differs from 1—the HQET prediction at leading order in $1/m_Q$. The deviation is due to the large recoil of the heavy quark against the brown muck in the charm system where $m_N/m_{\bar{D}} \approx 1/2$.

At leading order, the values of the heavy baryon observables, eqs (7), (8), (13), (14), (18), (19) and (20), depend on one parameter— κ . This parameter can be eliminated at LO in a model-independent way by using the mass splitting between the ground state and the first orbitally excited state of Λ_c in eq. (7). At LO, the model independent predictions are given in Table. I. The theoretical uncertainty of these model-independent predictions can be large; corrections are nominally of relative order $\lambda^{1/2} \sim (1/3)^{1/2} \sim 60\%$. To improve accuracy, the next-to-leading order corrections must be included. The predictions at NLO contain theoretical uncertainty of relative order λ which is nominally $\sim 1/3 \sim 30\%$.

At NLO, heavy baryon observables, eqs. (7), (8), (13), (14), (18), (19) and (20), contain an additional constant— α . Model-independent predictions at this order can, in principle, be made by eliminating κ and α in terms of any pair of observables. In order to experimentally test these predictions apart from the presently known value of the excitation energy of Λ_c ($\Delta m_c \approx 330 \text{ MeV}$), another observable (eg. a derivative of the semileptonic decay rate, ρ or σ , or the total radiative decay rate of the first excited state of Λ_c) has to be measured. The sensitivity of different observables

TABLE I. Model-independent predictions at LO. The observables are defined in eqs. (7), (8), (13), (14), (18), (19) and (20). Corrections are of relative order $\lambda^{1/2}$.

Fitted Observable	Predictions					
	$\Delta m_b (MeV)$	Θ_0	$\rho (MeV^{-3/2})$	$\sigma (MeV^{-3/4})$	$\Gamma_c (MeV)$	$\Gamma_b (MeV)$
$\Delta m_c \approx 330 MeV$	300	≈ 1	-1.2×10^{-4}	0.011	0.025	0.13

to the value of α varies. One therefore has to judiciously choose an additional observable. The next-to-leading order contribution to such observables should be large enough numerically so as to be stable against the NNLO corrections. Based on the sensitivity to α , we will argue that the best observables (in addition to Δm_c) are either the second derivative of the form factor $\Theta(z)$ at zero recoil— ρ —or the slope of the form factor $\Xi(z)$ at zero recoil— σ —or the total radiative decay rate of the first excited state of Λ_c or Λ_b — Γ_c or Γ_b .

To estimate the size of the NLO corrections we can rescale the constants κ and α and the observables in eqs. (7), (8), (13), (14), (18), (19) and (20) using the typical momentum scale of the collective degrees of freedom. It is natural to define this scale by,

$$\Lambda \equiv (\mu_c \Delta m_c^2)^{1/3} \approx 410 MeV, \quad (21)$$

with Λ^3 being equal to the value of κ determined from the leading order term in eq. (7): $\kappa = \mu_c \Delta m_c^2 (1 + \mathcal{O}(\lambda^{1/2}))$. In terms of the scale Λ , dimensionless constants $\bar{\kappa}$ and $\bar{\alpha}$ can be defined by,

$$\bar{\kappa} \equiv \Lambda^{-3} \kappa, \quad \bar{\alpha} \equiv \Lambda^{-5} \alpha. \quad (22)$$

The dimensionless constant $\bar{\kappa}$ is unity when only leading order terms in the combined expansion of the observables in eqs. (7), (8), (13), (14), (18), (19) and (20) are kept. When NLO terms are included, the constant $\bar{\kappa}$ has the form:

$$\bar{\kappa} = (1 + \delta\bar{\kappa}) (1 + \mathcal{O}(\lambda)), \quad (23)$$

where $\delta\bar{\kappa}$ is of relative order $\lambda^{1/2}$. Hence, when working up to NLO, the LO terms containing $\bar{\kappa}$ may be linearized with respect to $\delta\bar{\kappa}$. Moreover, since in all of the observables in eqs. (7), (8), (13), (14), (18), (19) and (20) the constants κ and α contribute only via the ratio α/κ^n (where n is an integer or half-integer), the value of $\bar{\kappa}$ in these expressions can be taken to be unity when NNLO corrections are neglected. In addition to using dimensionless constants $\bar{\kappa}$ and $\bar{\alpha}$, we can rescale the observables according to their dimensions using appropriate powers of Λ . The corresponding dimensionless observables are given by,

$$\begin{aligned} \Delta \bar{m}_c &\equiv \Delta m_c \Lambda^{-1} = (1 + \frac{1}{2} \delta\bar{\kappa}) \sqrt{\frac{\Lambda^3}{\mu_c}} + \frac{5}{4!} \frac{\bar{\alpha} \Lambda^2}{\mu_c} + \mathcal{O}(\lambda^{3/2}) \\ &\approx (0.80 (1 + \frac{1}{2} \delta\bar{\kappa}) + 0.13 \bar{\alpha}) (1 + \mathcal{O}(\lambda)), \\ \Delta \bar{m}_b &\equiv \Delta m_b \Lambda^{-1} = (1 + \frac{1}{2} \delta\bar{\kappa}) \sqrt{\frac{\Lambda^3}{\mu_b}} + \frac{5}{4!} \frac{\bar{\alpha} \Lambda^2}{\mu_b} + \mathcal{O}(\lambda^{3/2}) \\ &\approx (0.72 (1 + \frac{1}{2} \delta\bar{\kappa}) + 0.11 \bar{\alpha}) (1 + \mathcal{O}(\lambda)), \\ \bar{\Theta}_0 &\equiv \Theta_0 = \frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left(1 + \frac{\bar{\alpha} \sqrt{\Lambda}}{4!} \frac{45(\sqrt{\mu_b} - \sqrt{\mu_c})^2}{16\sqrt{\mu_b \mu_c}(\sqrt{\mu_b} + \sqrt{\mu_c})} \right) (1 + \mathcal{O}(\lambda)) \\ &\approx (0.998 + 5.6 \times 10^{-4} \bar{\alpha}) (1 + \mathcal{O}(\lambda)), \\ \bar{\rho} &\equiv \rho \Lambda^{3/2} = -\frac{2\sqrt{2} \mu_b^{3/8} \mu_c^{3/8}}{\Lambda^{3/2}(\sqrt{\mu_b} + \sqrt{\mu_c})^{3/2}} \left(1 - \frac{1}{2} \delta\bar{\kappa} + \frac{\bar{\alpha} \sqrt{\Lambda}}{4!} \frac{45(\sqrt{\mu_b} - \sqrt{\mu_c})^2 - 160\sqrt{\mu_b \mu_c}}{16\sqrt{\mu_b \mu_c}(\sqrt{\mu_b} + \sqrt{\mu_c})} \right) (1 + \mathcal{O}(\lambda)) \\ &\approx (-0.998 (1 - \frac{1}{2} \delta\bar{\kappa}) + 0.15 \bar{\alpha}) (1 + \mathcal{O}(\lambda)), \\ \bar{\sigma} &\equiv \sigma \Lambda^{3/4} = \frac{4 \mu_b^{3/8} \mu_c^{5/8}}{\Lambda^{3/4}(\sqrt{\mu_b} + \sqrt{\mu_c})^2} \left(1 - \frac{1}{4} \delta\bar{\kappa} + \frac{\bar{\alpha}}{4!} \frac{105\mu_b - 230\sqrt{\mu_b \mu_c} + 45\mu_c}{16\Lambda^{1/2}\sqrt{\mu_b \mu_c}(\sqrt{\mu_b} + \sqrt{\mu_c})} \right) (1 + \mathcal{O}(\lambda)) \\ &\approx (0.969 (1 - \frac{1}{4} \delta\bar{\kappa}) - 0.07 \bar{\alpha}) (1 + \mathcal{O}(\lambda)), \end{aligned}$$

TABLE II. Model-independent predictions at NLO. Observables are given in eqs. (7), (8), (13), (14), (18), (19) and (20). The typical scale Λ is defined as $\Lambda \equiv (\mu_c \Delta m_c^2)^{1/3} \approx 410 \text{ MeV}$. Corrections are of relative order λ .

Predictions	Fitted Observables		
	Δm_c and ρ	Δm_c and σ	Δm_c and Γ_c
$\Delta m_b \Lambda^{-1}$	$1.29 + 0.57 (\rho \Lambda^{3/2})$	$0.77 - 0.05 (\sigma \Lambda^{3/4})$	$0.71 + 0.27 (\Gamma_c \Lambda^{-1})$
Θ_0	$0.95 - 0.05 (\rho \Lambda^{3/2})$	$0.99 + 0.004 (\sigma \Lambda^{3/4})$	$0.999 - 0.02 (\Gamma_c \Lambda^{-1})$
$\rho \Lambda^{3/2}$		$-0.92 - 0.08 (\sigma \Lambda^{3/4})$	$-1.01 + 0.46 (\Gamma_c \Lambda^{-1})$
$\sigma \Lambda^{3/4}$	$0.25 - 0.72 (\rho \Lambda^{3/2})$		$0.98 - 0.33 (\Gamma_c \Lambda^{-1})$
$\Gamma_c \Lambda^{-1}$	$(2.67 + 2.62 (\rho \Lambda^{3/2})) \times 10^{-3}$	$(2.67 - 2.15 (\sigma \Lambda^{3/4})) \times 10^{-4}$	
$\Gamma_b \Lambda^{-1}$	$(1.27 + 1.24 (\rho \Lambda^{3/2})) \times 10^{-2}$	$(1.30 - 1.02 (\sigma \Lambda^{3/4})) \times 10^{-3}$	$(0.029 + 5.77 (\Gamma_c \Lambda^{-1})) \times 10^{-3}$

$$\begin{aligned}
\bar{\Gamma}_c &\equiv \frac{6}{e^2} \Gamma_c \Lambda^{-1} = \left(\frac{m_{\bar{D}} - m_N}{m_{\bar{D}} m_N} \right)^2 \Lambda^3 \left(1 + \delta \bar{\kappa} - \frac{\bar{\alpha} \sqrt{\Lambda}}{4!} \frac{5}{\sqrt{\mu_c}} \right) (1 + \mathcal{O}(\lambda)) \\
&\approx (0.05 (1 + \delta \bar{\kappa}) - 0.01 \bar{\alpha}) (1 + \mathcal{O}(\lambda)) , \\
\bar{\Gamma}_b &\equiv \frac{6}{e^2} \Gamma_b \Lambda^{-1} = \left(\frac{m_{\bar{B}} + m_N}{m_{\bar{B}} m_N} \right)^2 \Lambda^3 \left(1 + \delta \bar{\kappa} - \frac{\bar{\alpha} \sqrt{\Lambda}}{4!} \frac{5}{\sqrt{\mu_b}} \right) (1 + \mathcal{O}(\lambda)) \\
&\approx (0.26 (1 + \delta \bar{\kappa}) - 0.04 \bar{\alpha}) (1 + \mathcal{O}(\lambda)) ,
\end{aligned} \tag{24}$$

where we linearized the terms containing $\bar{\kappa}$. In the NLO terms this amounts to taking $\bar{\kappa}$ to be 1 as was discussed above. In the second set of equalities in eq. (24) the experimental values for the masses have been used.

The rescaled observables in eq. (24) depend linearly on the dimensionless constants $\delta \bar{\kappa}$ and $\bar{\alpha}$. The value of $\bar{\alpha}$ is expected to be of order unity. The NLO corrections for all observables in eq. (24), except for Θ_0 , are 7% to 20% when $\bar{\alpha}$ is taken to be unity. The NLO contributions to the value of Θ_0 is suppressed by four orders of magnitude. The leading order value of Θ_0 is very close to unity (see Table I)—the leading order value predicted by HQET. The deviation from unity is an exceptionally poor way to extract α since the sensitivity is so small numerically that one expects higher order terms to be significant. The sensitivities with respect to $\bar{\kappa}$ can be used to decide which observables are best suited to extract values of $\bar{\kappa}$ and $\bar{\alpha}$.

Formally, the $1/m_Q$ and $1/N_c$ corrections in the combined expansion are of the same order. However, the heavy quark expansion is generically expected to be more an accurate approximation for heavy baryons than the large N_c expansion. If Δm_b is used apart from Δm_c to eliminate κ and α , the resulting relations contain mass differences, $\Delta m_c - \Delta m_b$. In the heavy quark limit this difference is zero up to $1/m_Q$ corrections. At leading order in the combined expansion this difference is approximately 30 MeV (Table I). The NNLO corrections which are beyond the accuracy we are working to can be estimated from the mass splitting of the states of the doublet (Λ_{c1} , Λ_{c1}^*). This splitting—approximately 40 MeV —is the NNLO effect containing $1/m_Q$ and $1/N_c$ corrections. Hence, predictions containing $\Delta m_c - \Delta m_b$ and $\Delta m_c + \Delta m_b$ are sensitive to NNLO corrections and should not be used to extract κ and α .

Any of the remaining observables— ρ , σ and Γ_c (or Γ_b)—can be used along with Δm_c to eliminate κ and α . Using $\bar{\Delta m}_c$ and $\bar{\rho}$ as primary observables, we can obtain model-independent predictions for other observables. These predictions are listed in the second column in Table II. Similarly, using eq. (24), $\bar{\kappa}$ and $\bar{\alpha}$ can be extracted from pairs ($\bar{\Delta m}_c$, $\bar{\sigma}$) and (Δm_c , $\bar{\Gamma}_c$). Corresponding model-independent relations are given in the third and the fourth columns in Table II. The corrections to relations in Table II are generically of order λ which nominally is $1/3 \sim 30\%$. In order to test NLO predictions, measurements of one of the observables—the curvature of the form factor $\Theta(z)$ at zero recoil, the slope of the form factor $\Xi(z)$ at zero recoil, or the total radiative decay rate of the first excited state of Λ_c —has to be measured.

We have discussed the model-independent relations between observables which arise in the combined heavy quark and large N_c expansion at leading and next-to-leading order. These observables were derived in the framework of an effective theory in the combined heavy quark and large N_c limit in [16,17]. At leading order in the combined expansion a model independent expressions for a number of heavy baryon observables contain one constant. This coefficient can be eliminated using the known value of the excitation energy of the first excited state of Λ_c , Δm_c . The resulting predictions up to corrections of order $\lambda^{1/2}$ are listed in Table I. To obtain predictions at next-to-leading order in the combined expansion, an additional observable is required. At the present time the data is not available. Observables in eqs. (7), (8), (13), (14), (18), (19) have different sensitivities with respect to the next-to-next-to-leading order corrections. We have discussed observables (when data becomes available) should be used in addition to the excitation energy Δm_c to predict the remaining observables. The model-independent relations at NLO are given in Table II.

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